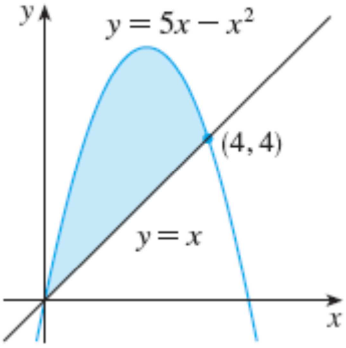
7.1

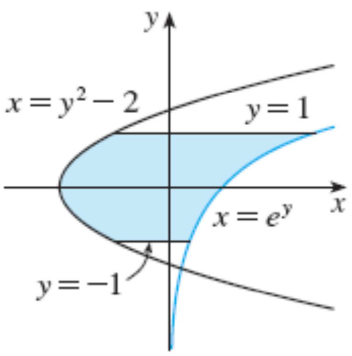
1-4. Find the area of the shaded region.

1.



$$A=\int_0^4\left[(5x-x^2)-x\right]dy=2x^2-\frac{x^3}{3}\bigg|_0^4=\frac{32}{3}$$

3.



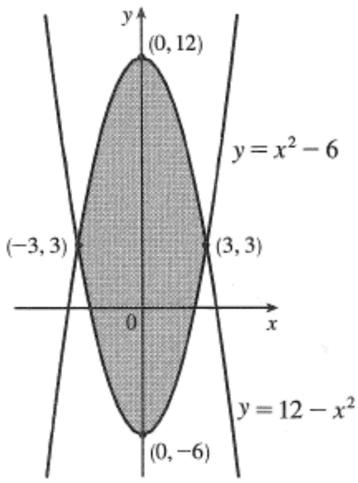
$$A=\int_{-1}^1\left[e^y-(y^2-2)\right]dy=e^y-\frac{1}{3}y^3+2y\bigg|_{-1}^1=e-\frac{1}{e}+\frac{10}{3}$$

5-16. Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

9. $y=12-x^2, y=x^2-6$

先找出兩曲線交點作為積分上下限，$2x^2=18\Rightarrow x=-3,3$，

則 $A=\int_{-3}^3\left[(12-x^2)-(x^2-6)\right]dx=2\left[18x-\frac{2}{3}x^3\right]_0^3=72$

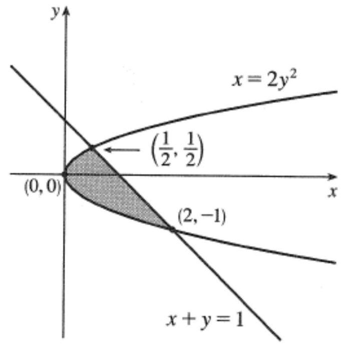


11. $x=2y^2, x+y=1$

先找出兩曲線交點作為積分上下限，

$2y^2+y-1=0 \Rightarrow y=\frac{1}{2},-1$，

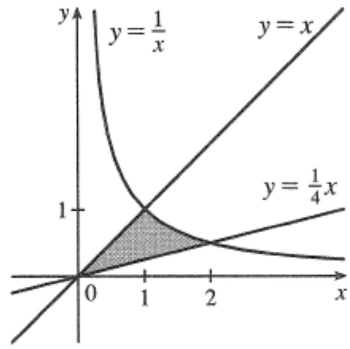
則 $A=\int_{-1}^{1/2}\left[(1-y)-2y^2\right]dy = y-\frac{1}{2}y-\frac{2}{3}y^3\Big|_{-1}^{1}=\frac{9}{8}$



15. $y=\frac{1}{x}, y=x, y=\frac{x}{4}, x>0$

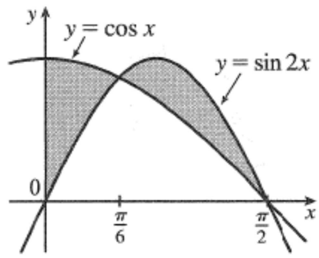
先找出各曲線交點作為圖形邊點，如右圖

則 $A=\int_0^1 (x-\frac{1}{4}x)dx+\int_1^2 (\frac{1}{x}-\frac{1}{4}x)dx=\frac{3}{8}x^2\Big|_0^1+\left[\ln|x|-\frac{x^2}{8}\right]_1^2=\ln 2$



21. Sketch the region that lies between the curves $y=\cos x$ and $y=\sin 2x$ and between $x=0$ and $x=\frac{\pi}{2}$. Notice that the region consists of two separate parts. Find the area of this region.

$$A=\int_0^{\pi/6}(\cos x-\sin 2x)dx+\int_{\pi/6}^{\pi/2}(\sin 2x-\cos x)dx$$
$$=\left[\sin x+\frac{1}{2}\cos 2x\right]_0^{\pi/6}+\left[-\frac{1}{2}\cos 2x-\sin x\right]_{\pi/6}^{\pi/2}=\frac{1}{2}$$



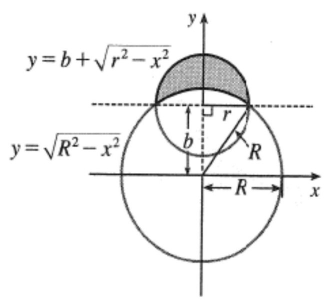
29. Find the area of the crescent-shaped region (called a *lune*) bounded by arcs of circles with radii r and R(see the figure).

由圖可看出，該斜線面積對y軸對稱，所以

$$A=2\int_0^r (b+\sqrt{r^2-x^2}-\sqrt{R^2-x^2})dx$$
$$=2br+2\int_0^r \sqrt{r^2-x^2}dx+2\int_0^r \sqrt{R^2-x^2}dx$$

其中，令 $x=r\sin t \Rightarrow dx=r\cos t dt$，

則 $\int_0^r \sqrt{r^2-x^2}dx=r^2\int_0^{\pi/2}\cos^2 t dt=r^2\left[\frac{1}{2}t+\frac{1}{4}\sin 2t\right]_0^{\pi/2}=\frac{\pi r^2}{4}$



令 $x=R\sin t \Rightarrow dx=R\cos t dt$，

廖彥法 整理

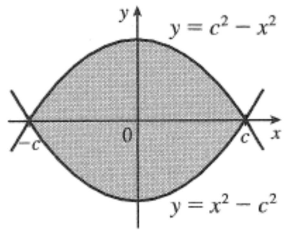
則 $\int_0^r \sqrt{R^2-x^2}\,dx = R^2\int_0^{\sin^{-1}(r/R)} \cos^2 t\,dt = R^2\left[\frac{1}{2}t+\frac{1}{4}\sin 2t\right]_0^{\sin^{-1}(r/R)} = \frac{1}{2}R^2\sin^{-1}\frac{r}{R}+\frac{r}{2}\sqrt{R^2-r^2}$

且 $b=\sqrt{R^2-r^2}$

所以 $A = r\sqrt{R^2-r^2}+\frac{\pi r^2}{2}-R^2\sin^{-1}\frac{r}{R}$

31. Find the values of c such that the area of the region bounded by the parabolas $y=x^2-c^2$ and $y=c^2-x^2$ is 576.

兩函數交點在 $x=\pm c$，且圖形對於x軸，y軸對稱，

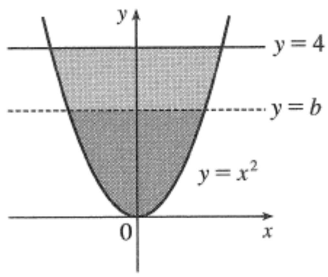


故 $A = 4\int_0^c (c^2-x^2)dx = 4(c^2x-\frac{x^3}{3})\Big|_0^c = \frac{8}{3}c^3 = 576 \Rightarrow c=6$

且 $c=-6$ 亦會是一解。

33. Find the number b such that the line $y=b$ divides the region bounded by the curves $y=x^2$ and $y=4$ into two regions with equal area.

由圖可知對稱於y軸，故



$A = 2\int_0^2 (4-x^2)dx = 2A_b \Rightarrow A_b = [4x-\frac{x^3}{3}]_0^2 = \frac{16}{3}$

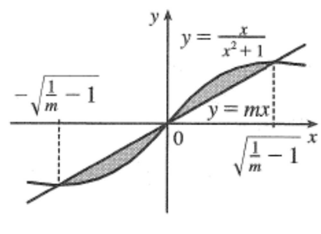
且 $A_b = 2\int_0^{\sqrt{b}} (b-x^2)dx = \frac{4}{3}b^{3/2} \Rightarrow b = 4^{2/3} \approx 2.52$

37. For what values of m do the line $y=mx$ and the curve $y=\frac{x}{x^2+1}$ enclose a region? Find the area of the region.

由圖可知兩曲線相交時，交點 $x=\pm\sqrt{\frac{1}{m}-1}$ 存在，$\frac{1}{m}>1 \Rightarrow 0<m<1$

則相交的面積亦對稱於原點，故

$A = 2\int_0^{\sqrt{\frac{1}{m}-1}}\left(\frac{x}{x^2+1}-mx\right)dx = \left[\ln|x^2+1|-mx^2\right]_0^{\sqrt{\frac{1}{m}-1}} = m-\ln m-1$

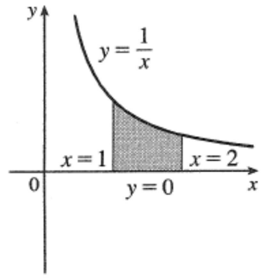


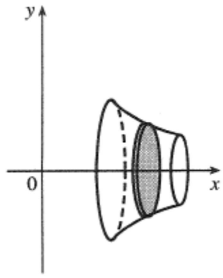
7.2

1-12. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

1. $y = 1/x, x = 1, x = 2, y = 0;$ *about the* $x-axis$

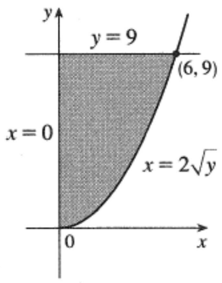
$$V = \pi\int y^2 dx = \pi\int_1^2 \frac{1}{x^2}dx = -\frac{\pi}{x}\Bigg|_1^2 = \frac{\pi}{2}$$

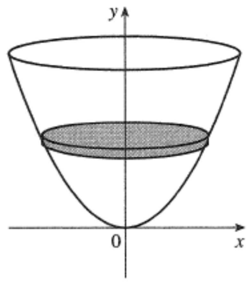




3. $x = 2\sqrt{y}, x = 0, y = 9;$ *about the* $y-axis$

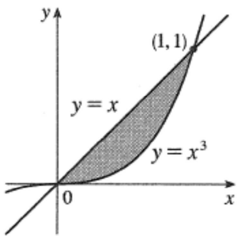
$$V = \pi\int x^2 dy = 4\pi\int_0^9 y dx = 2\pi y^2\Big|_0^9 = 162\pi$$

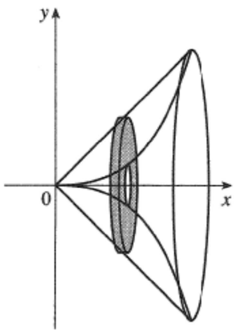




5. $y = x^3, y = x, x \ge 0;$ *about the* $x-axis$

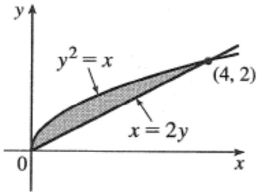
$$V = \pi\int (y_1^{\ 2} - y_2^{\ 2})dx = \pi\int_0^1 (x^2 - x^6)dx = \pi(\frac{1}{3}x^3 - \frac{1}{7}x^7)\Bigg|_0^1 = \frac{4\pi}{21}$$

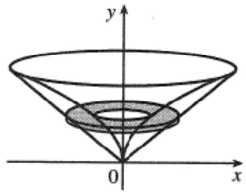




7. $y^2 = x, x = 2y;$ *about the* $y-axis$

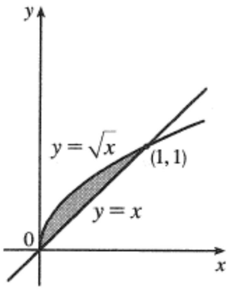
$$V = \pi\int (x_1^{\ 2} - x_2^{\ 2})dy = \pi\int_0^2 (4y^2 - y^4)dy = \pi(\frac{4}{3}y^3 - \frac{1}{5}y^5)\Bigg|_0^2 = \frac{64\pi}{15}$$

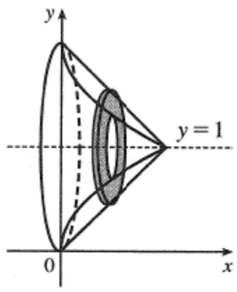




9. $y = x,\ y = \sqrt{x};\ about\ y = 1$

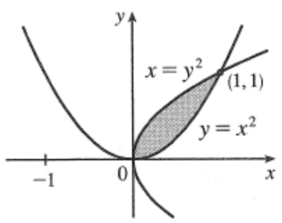
$$V = \pi\int (y_1^{\ 2} - y_2^{\ 2})dx = \pi\int_0^1 [(1-x)^2 - (1-\sqrt{x})^2]dx = \pi(\frac{-3}{2}x^2 + \frac{1}{3}x^3 + \frac{4}{3}x^{3/2})\Big|_0^1 = \frac{\pi}{6}$$

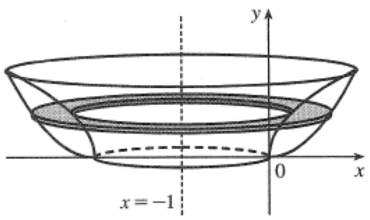




11. $y = x^2,\ x = y^2;\ about\ x = -1$

$$V = \pi\int (x_1^{\ 2} - x_2^{\ 2})dy = \pi\int_0^1 [(\sqrt{y}+1)^2 - (y^2+1)^2]dy = \pi(\frac{1}{2}y^2 + \frac{4}{3}y^{3/2} - \frac{1}{5}y^5 - \frac{2}{3}y^3)\Big|_0^1 = \frac{29\pi}{30}$$

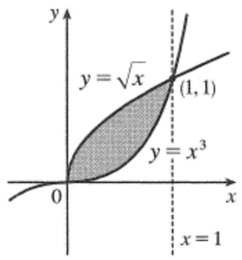




13. The region enclosed by the curves $y = x^3$ and $y = \sqrt{x}$ is rotated about the line $x = 1$. Find the volume of the resulting solid.

$$V = \pi\int (x_1^{\ 2} - x_2^{\ 2})dy = \pi\int_0^1 [(1-y^2)^2 - (1-\sqrt[3]{y})^2]dy$$

$$= \pi(-\frac{2}{3}y^3 + \frac{1}{5}y^5 + \frac{3}{2}y^{4/3} - \frac{3}{5}y^{5/3})\Big|_0^1 = \frac{13\pi}{30}$$



21. Each integral represents the volume of a solid. Describe the solid.

(a) $\pi\int_0^{\pi/2} \cos^2 x dx$ (b) $\pi\int_0^1 (y^4 - y^8)dy$

(a) 為 $y = \cos x, 0 \le x \le \frac{\pi}{2}$ 對x軸旋轉所得體積。

(b)為 $y = \sqrt{x}$ 與 $y = \sqrt[4]{x}$ 所交會的區域對y軸旋轉所得體積。

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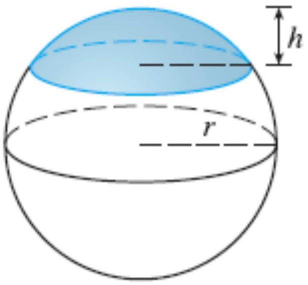
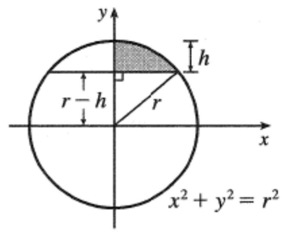
25-37. Find the volume of the described solid S.

27. A cap of a sphere with radius *r* and height *h*.





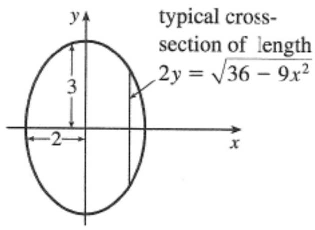
因為為圓形，故 $x^2+y^2=r^2 \Rightarrow x^2=r^2-y^2$ ，$r-h\le y\le r$

$$V=\pi\int x^2dy=\pi\int_{r-h}^{r}(r^2-y^2)dx=\pi\left[r^2y-\frac{1}{3}y^3\right]_{r-h}^{r}=\pi h^2(r-\frac{h}{3})$$

33. The base of *S* is an elliptical region with boundary curve $9x^2+4y^2=36$. Cross-sections perpendicular to the *x*-axis are isosceles right triangles with hypotenuse in the base.

$9x^2+4y^2=36\Rightarrow y=\pm\dfrac{\sqrt{36-9x^2}}{2}$,$-2\le x\le 2$ 取正，為橢圓上半部的曲線函數

$$V=\pi\int y^2dx=\frac{\pi}{4}\int_{-2}^{2}(36-9x^2)dx=\pi\left[9x-\frac{3}{4}x^3\right]_{-2}^{2}=24\pi$$



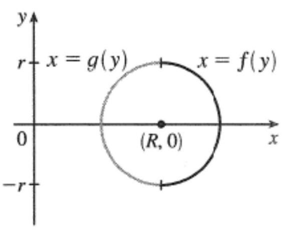
41. (a) Set up an integral for the volume of a solid *torus* (the donut-shaped solid shown in the figure) with radii *r* and *R*.

(b) By interpreting the integral as an area, find the volume of the torus.

(a) 該圓形函數為 $(x-R)^2+y^2=r^2$

分成左右兩邊的函數：左函數 $x_L=R-\sqrt{r^2-y^2}$ ；右函數 $x_R=R+\sqrt{r^2-y^2}$ ，$0\le y\le r$

$$V=\pi\int(x_R{}^2-x_L{}^2)dy=8\pi R\int_0^r\sqrt{r^2-y^2}dy$$



(b) 令 $y=r\sin\theta\Rightarrow dy=r\cos\theta d\theta$

則 $8\pi R\int_0^r\sqrt{r^2-y^2}dy=8\pi Rr^2\int_{\pi/2}^{0}\cos^2\theta d\theta=8\pi Rr^2\cdot\frac{1}{4}\pi=2\pi^2r^2R$

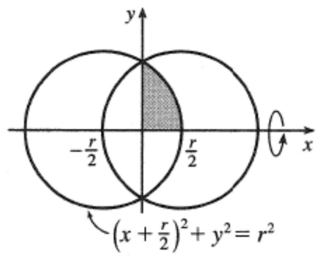
45. Find the volume common to two spheres, each with radius r, if the center of each sphere lies on the surface of the other sphere.

左邊圓形函數為 $(x+\frac{r}{2})^2+y^2=r^2$

而斜線部分的函數為 $y=\sqrt{r^2-(x+\frac{r}{2})^2}$ ， $0\le x\le\frac{r}{2}$

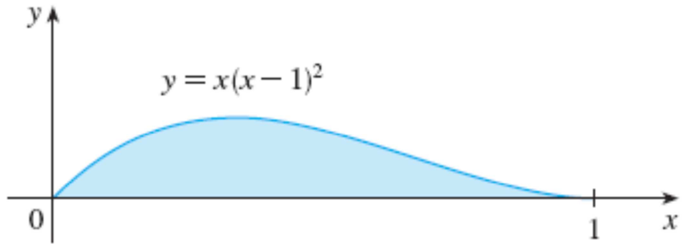
$$V=\pi\int y^2dx=\pi\int_0^{r/2}(\frac{3}{4}r^2-x^2-xr)dx=\pi(\frac{3}{4}r^2x-\frac{1}{3}x^3-\frac{1}{2}x^2r)\Big|_0^{r/2}=\frac{5}{24}\pi r^3$$

其全部體積為斜線部分對x軸旋轉體積的兩倍，即 $\frac{5}{12}\pi r^3$



7.3

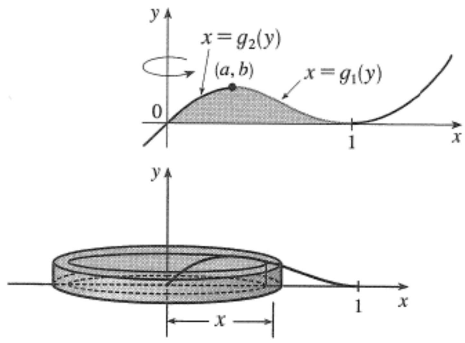
1. Let S be the solid obtained by rotating the region shown in the figure about the y-axis. Explain why it is awkward to use slicing to find the volume V of S. Sketch a typical approximating shell. What are its circumference and height? Use shells to find V.



若使用圓環法(Washer)，則必須找出 $y=x(x-1)^2$ 的頂點，將其分成左右兩邊的函數，以進行體積積分，對於該函數而言，難以化整為好積分的函數。

若使用圓柱殼法(Shell Method)，

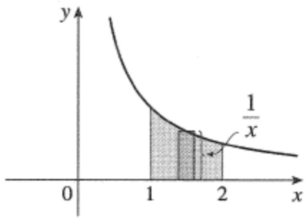
$$V=2\pi\int xy\,dx=2\pi\int_0^1 x^2(x-1)^2dx=2\pi\left[\frac{x^5}{5}-\frac{x^4}{2}+\frac{x^3}{3}\right]_0^1=\frac{\pi}{15}$$

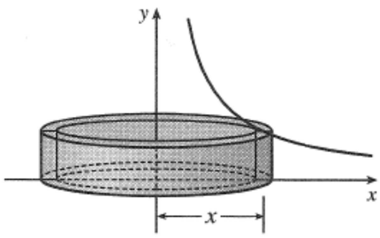


3-7. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis. Sketch the region and a typical shell.

3. $y=1/x, y=0, x=1, x=2$

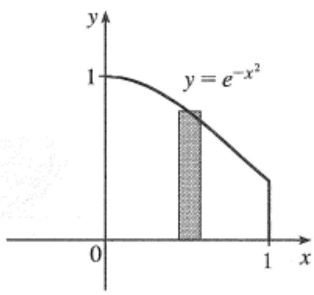
$$V=2\pi\int xy\,dx=2\pi\int_1^2 x\cdot\frac{1}{x}dx=2\pi$$

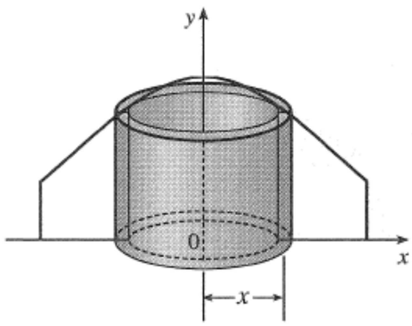




5. $y=e^{-2x^2}, y=0, x=0, x=1$

$$V=2\pi\int xy\,dx=2\pi\int_0^1 x\cdot e^{-2x^2}dx=-\pi e^{-x^2}\Big|_0^1=\pi(1-e^{-1})$$



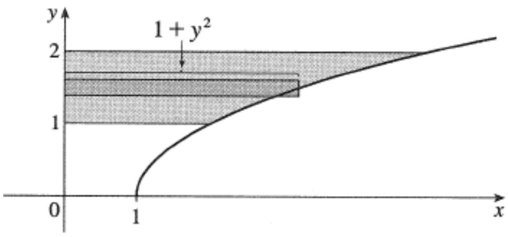


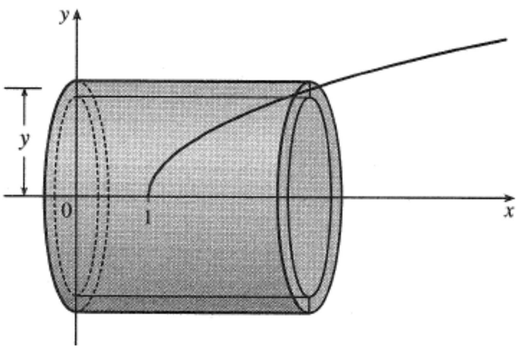
廖彥法 整理

9-14. Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis. Sketch the region and a typical shell.

9. $x = 1 + y^2, x = 0, y = 1, y = 2$

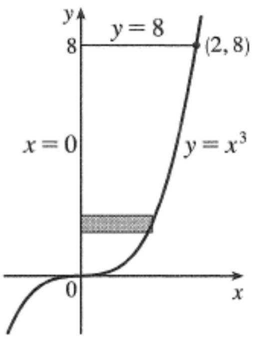
$$V = 2\pi \int xydy = 2\pi \int_1^2 (1 + y^2) \cdot ydy = 2\pi (\frac{1}{2} y^2 + \frac{1}{4} y^4) \Big|_1^2 = \frac{21\pi}{2}$$

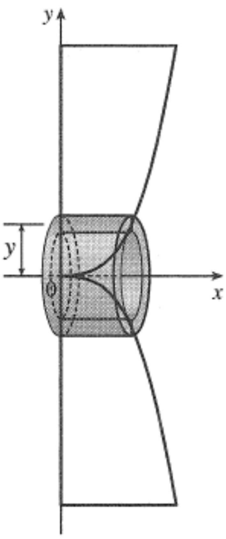




11. $y = x^3, y = 8, x = 0$

$$V = 2\pi \int xydy = 2\pi \int_0^8 \sqrt[3]{y} \cdot ydy = \frac{6}{7}\pi y^{7/3} \Big|_0^8 = \frac{768\pi}{7}$$





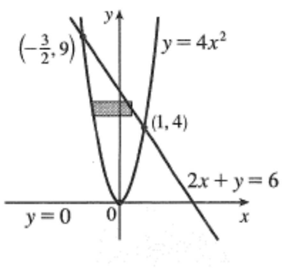
13. $y = 4x^2, 2x + y = 6$

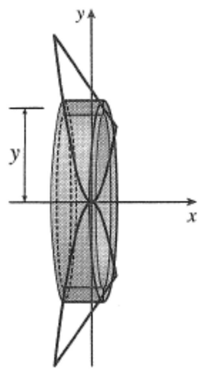
先找出兩函數交點

$$\begin{cases} y = 4x^2 \\ 2x + y = 6 \end{cases} \Rightarrow x = \frac{-3}{2}, 1 \Rightarrow y = 9, 4$$

$$V = 2\pi \int y(x_1 - x_2)dy = 2\pi \int_0^4 y[\frac{\sqrt{y}}{2} - (-\frac{\sqrt{y}}{2})]dy + 2\pi \int_4^9 y[(-\frac{y}{2} + 3) - (-\frac{\sqrt{y}}{2})]dy$$

$$= \frac{4}{5}\pi y^{5/2} \Big|_0^4 + 2\pi \left[-\frac{y^3}{6} + \frac{3}{2} y^2 + \frac{1}{5} y^{5/2} \right]_4^9 = \frac{250}{3}\pi$$

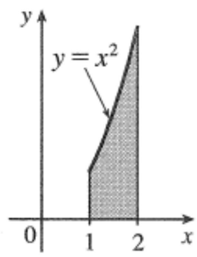


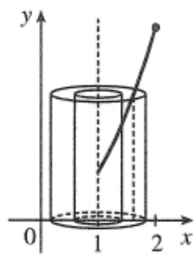


15-20. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis. Sketch the region and a typical shell.

15. $y = x^2, y = 0, x = 1, x = 2; about\ x = 1$

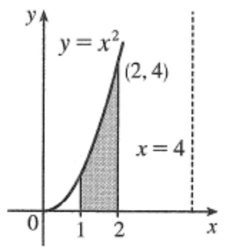
$$V = 2\pi\int (x-1)y dx = 2\pi\int_1^2 (x-1)x^2 dx = 2\pi\left[\frac{x^4}{4} - \frac{x^3}{3}\right]_1^2 = \frac{17}{6}\pi$$

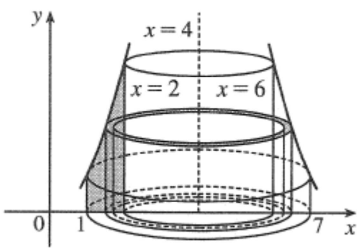




17. $y = x^2, y = 0, x = 1, x = 2; about\ x = 4$

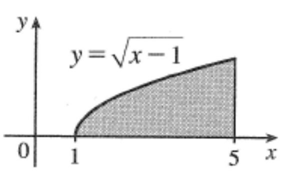
$$V = 2\pi\int (4-x)y dx = 2\pi\int_1^2 (4-x)x^2 dx = 2\pi\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_1^2 = \frac{67}{6}\pi$$

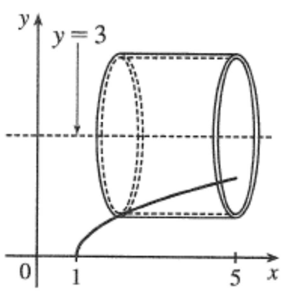




19. $y = \sqrt{x-1}, y = 0, x = 5; about\ y = 3$

$$V = 2\pi\int (3-y)(5-x)dy = 2\pi\int_0^2 (3-y)(5-y^2-1)dy = 2\pi\left[12y - 2y^2 - y^3 + \frac{y^4}{4}\right]_0^2 = 24\pi$$

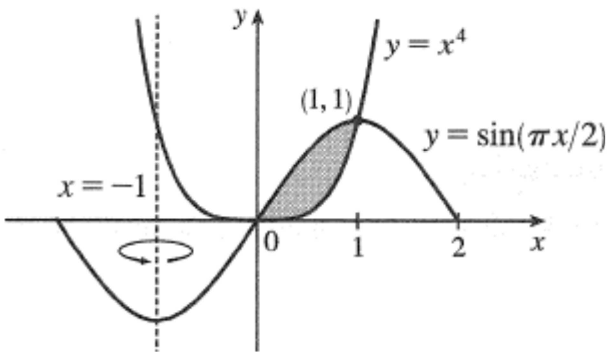




21-26. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

23. $y = x^4, y = \sin(\pi x/2); about\ x = -1$

$$V = 2\pi \int x(y_1 - y_2)dx = 2\pi \int_0^1 (x+1)(\sin\frac{\pi}{2}x - x^4)dx$$



29-32. Each integral represents the volume of a solid. Describe the solid.

29. $\int_0^3 2\pi x^5 dx$

$\int_0^3 2\pi x^5 dx = 2\pi \int_0^3 x \cdot x^4 dx$，即 $0 \le y \le x^4$，$0 \le x \le 3$ 之區域對 y 軸旋轉所得之體積。

31. $\int_0^1 2\pi(3-y)(1-y^2)dy$

$\int_0^1 2\pi(3-y)(1-y^2)dy = 2\pi \int_0^1 (y-3)(y^2-1)dy$，即(i) $x = y^2 - 1$、$x = 0$、$y = 0$ 所圍成之區域對 $y = 3$ 軸旋轉所得之體積。(ii) $x = y^2$、$x = 1$、$y = 0$ 所圍成之區域對 $y = 3$ 軸旋轉所得之體積。

33-38. The region bounded by the given curves is rotated about the specified axis. Find the volume of the resulting solid by any method.

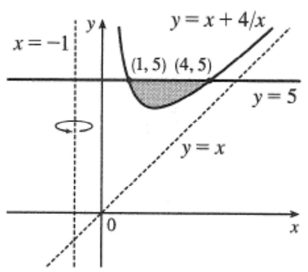
33. $y = x^2 + x - 2, y = 0; about\ x - axis$

$y = x^2 + x - 2, y = 0 \Rightarrow x = -2, 1$

$$V = \pi \int y^2 dx = \pi \int_{-2}^1 (x^2 + x - 2)^2 dx = \pi \left[\frac{x^5}{5} + \frac{x^4}{2} - x^3 - 2x^2 + 4x\right]_{-2}^1 = \frac{81}{10}\pi$$

35. $y = 5, y = x + (4/x); about\ x = -1$

$$V = 2\pi \int_1^4 [x - (-1)][5 - (x + \frac{4}{x})]dx = 2\pi \left[-\frac{x^3}{3} + 2x^2 + x - 4\ln x\right]_1^4 = 8\pi(3 - \ln 4)$$



 廖彥法 整理

7.4

1. Use the arc length formula (3) to find the length of the curve $y=2-3x, -2\le x\le 1$. Check your answer by noting that the curve is a line segment and calculating its length by the distance formula.

$$L=\int\sqrt{1+(y')^2}dx=\int_{-2}^{1}\sqrt{1+(-3)^2}dx=3\sqrt{10}$$

對於線段 $y=2-3x, -2\le x\le 1$，其端點為 $(-2,8),(1,-1)$，

所以長度為 $\sqrt{[1-(-2)]^2+[(-1)-8]^2}=3\sqrt{10}$

3-14. Find the length of the curve.

3. $y=1+6x^{3/2}, 0\le x\le 1$

$$L=\int\sqrt{1+(y')^2}dx=\int_0^1\sqrt{1+(9x^{1/2})^2}dx=\int_0^1\sqrt{1+81x}dx$$

令 $t=1+81x\Rightarrow dt=81dx$

則 $\int_0^1\sqrt{1+81x}dx=\frac{1}{81}\int_1^{82}t^{1/2}dt=\frac{2}{243}t^{3/2}\Big|_1^{82}=\frac{2}{243}(82\sqrt{82}-1)$

5. $y=\frac{x^5}{6}+\frac{1}{10x^3}, 1\le x\le 2$

$$L=\int\sqrt{1+(y')^2}dx=\int_1^2\sqrt{(\frac{5}{6}x^4+\frac{3}{10}x^{-4})^2}dx=\int_1^2\frac{5}{6}x^4+\frac{3}{10}x^{-4}dx=\left[\frac{1}{6}x^5-\frac{3}{10}x^{-3}\right]_1^2=\frac{1261}{240}$$

9. $y=\ln(\sec x), 0\le x\le \pi/4$

$$L=\int\sqrt{1+(y')^2}dx=\int_0^{\pi/4}\sqrt{(\sec x)^2}dx=\int_0^{\pi/4}\sec x dx=\ln(\sec x+\tan x)\Big|_0^{\pi/4}=\ln(\sqrt{2}+1)$$

13. $y=e^x, 0\le x\le 1$

$$L=\int\sqrt{1+(y')^2}dx=\int_0^1\sqrt{1+e^{2x}}dx$$

令 $\tan t=e^x\Rightarrow x=\ln|\tan t|\Rightarrow dx=\frac{1}{\sin t\cos t}dt$

則 $\int\sqrt{1+e^{2x}}dx=\int\frac{1}{\sin t\cos^2 t}dt$

令 $u=\cos t\Rightarrow du=-\sin t dt$

則 $\int\frac{1}{\sin t\cos^2 t}dt=\int\frac{1}{(u^2-1)u^2}du=\int\left(-\frac{1}{u^2}+\frac{1/2}{u-1}-\frac{1/2}{u+1}\right)du=\frac{1}{u}+\frac{1}{2}\ln|u-1|-\frac{1}{2}\ln|u+1|+C$

$$=\frac{1}{u}+\frac{1}{2}\ln\left|\frac{u-1}{u+1}\right|+C=\frac{1}{\cos t}+\frac{1}{2}\ln\left|\frac{\cos t-1}{\cos t+1}\right|+C$$

廖彥法 整理

$$=\sqrt{1+e^{2x}}+\frac{1}{2}\ln\left|\frac{\frac{1}{\sqrt{1+e^{2x}}}-1}{\frac{1}{\sqrt{1+e^{2x}}}+1}\right|+C=\sqrt{1+e^{2x}}+\frac{1}{2}\ln\left|\frac{1-\sqrt{1+e^{2x}}}{1+\sqrt{1+e^{2x}}}\right|+C$$

所以 $\int_0^1\sqrt{1+e^{2x}}\,dx=\left[\sqrt{1+e^{2x}}+\frac{1}{2}\ln\left|\frac{1-\sqrt{1+e^{2x}}}{1+\sqrt{1+e^{2x}}}\right|\right]_0^1$

$$=\sqrt{1+e^{2}}+\frac{1}{2}\ln\left|\frac{1-\sqrt{1+e^{2}}}{1+\sqrt{1+e^{2}}}\right|-\sqrt{2}-\frac{1}{2}\ln\left|\frac{1-\sqrt{2}}{1+\sqrt{2}}\right|$$

17. $x=y+y^3, 1\le y\le 4$,Set up, but do not evaluate, an integral for the length of the curve.

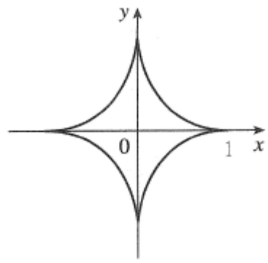
$x=y+y^3\Rightarrow\frac{dx}{dy}=1+3y^2$

$L=\int\sqrt{1+(x')^2}\,dy=\int_1^4\sqrt{2+6y^2+9y^4}\,dy$

25. Sketch the curve with equation $x^{2/3}+y^{2/3}=1$ and use symmetry to find its length.

由右圖可知其圖形對x軸、y軸、原點對稱，故僅需求第一象限的曲線即可，

$L=4\int_0^1\sqrt{1+(y')^2}\,dx=4\int_0^1 x^{-1/3}dx=4\lim_{t\to0^+}\left[\frac{3}{2}x^{2/3}\right]_t^1=6$



27. Find the arc length function for the curve $y=2x^{3/2}$ with starting point $P_0(1,2)$.

$y=2x^{3/2}\Rightarrow y'=3x^{1/2}$

$L(t)=\int\sqrt{1+(y')^2}\,dx=\int_1^t\sqrt{1+9x}\,dx$

令 $u=1+9x\Rightarrow du=9dx$

則 $\int_1^t\sqrt{1+9x}\,dx=\frac{1}{9}\int_{10}^{1+9t}u^{1/2}du=\frac{2}{27}u^{3/2}\Big|_{10}^{1+9t}=\frac{2}{27}\left[(1+9t)^{3/2}-10\sqrt{10}\right]$